HORIZONTAL IMPACT OF A FLOATING ELLIPSE ON AN INCOMPRESSIBLE FLUID

(GORIZONTAL'NYI UDAR PLAVAIUSHCHEGO ELLIPSA O NESZHIMAEMUIU ZHIDKOST')

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N.A. KUDRIAVTSEVA
(Moscow)

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In papers devoted to the impact of a body on a fluid, it is shown that in the absence of inertial forces the motion which arises after the impact is potential.

The impulsive pressure p_t is related to the velocity potential ϕ by the simple relation $p_t = -\rho\phi$, and the problem is reduced to solving the Laplace equation with the boundary condition $\phi = 0$ at the free surface of the fluid and $\partial\phi/\partial n = V_n$ at the wetted surface of the body, where V_n is the normal projection of the velocity of the body. The latter condition is correct if the fluid does not separate from the surface of the body. The assumption that the fluid does not separate immediately after the impact may lead to a solution which does not have any physical meaning, since in this case regions of negative impulsive pressure are obtained within the fluid [1].

An analytical solution of the problem of the horizontal impact of a half-submerged ellipse is given below, together with some experiments.

1. For the impulse and the impulsive moment which act on a body during the time of impact we have the dependence

$$J = J_y + iJ_z = i\int_\Sigma p_t\,ds = -i\rho\int_\Sigma \varphi\,ds \qquad (s = y + iz) \tag{1.1}$$

$$M = \int (y\,dZ - z\,dY) = -\operatorname{Re}\left\{\rho\int \bar{s}\varphi\,ds\right\}$$

where Y, Z are forces in the direction of the y and z axes.

To determine the impulsive forces and the moment due to the impact of the ellipse we shall find the characteristic function $w = \phi + i\psi$.

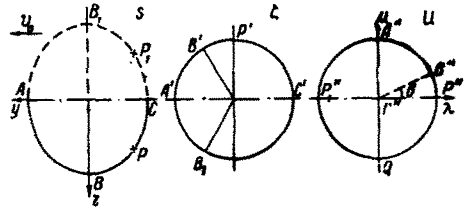

FIG. 1.

For its determination we have the following conditions (Fig. 1):

$$\varphi = 0 \quad \text{on } PC, \qquad \frac{\partial\varphi}{\partial n} = V_n \quad \text{on } ABP \quad \text{or} \quad \psi = v_0 z \tag{1.2}$$

where v_0 is the horizontal velocity which the ellipse acquires from the external impulse. Continuing the characteristic function into the upper half-plane we obtain

$$\varphi = 0 \quad \text{on } P_1C, \qquad \psi = -v_0 z \quad \text{on } AB_1P_1 \tag{1.3}$$

The boundary of the separated region is determined by the condition of continuous variation of the fluid velocity in this region and by the fact that the normal velocity of the body on the part of the surface of the ellipse on which the fluid separation occurs is greater than the normal velocity of the fluid. To approach the point of separation P from above along an arc of the ellipse one must satisfy the condition

$$\lim \frac{\partial\varphi}{\partial y} = v_0 \quad \text{or} \quad \lim \frac{\partial\psi}{\partial z} = v_0 \tag{1.4}$$

We shall map the region $s = y + iz$ external to the surface of the ellipse onto the upper interior of a single semi-circle in the plane $u = \lambda + i\mu$ so that the presently unknown points of separation P and P_1 transform into the ends of the actual diameter and the point C into the center of the semi-circle.

The function which realizes this conformal transformation has the form

$$s = y + iz = \frac{4au^2\cos\sigma_1 - a(u^2+1) - 4ibu(u^2+1)\cos\sigma_1}{(u^2+1)^2 + 4u^2\cos^2\sigma_1} \tag{1.5}$$

where a, b are the semi-axes of the ellipse, σ_1 is the argument of the point of separation P'' in the u-plane which is related to the ordinate of the point of separation

$$z_P = \frac{2b \cos \sigma_1}{1 + \cos^2 \sigma_1} \tag{1.6}$$

The boundary condition for the circular contour K of the plane $u = \lambda + i\mu$ transferred to the s-plane will be

$$\psi(\sigma) = v_0 \frac{2b \cos \sigma_1 \cos \sigma}{\cos^2 \sigma_1 + \cos^2 \sigma} \tag{1.7}$$

The required characteristic function, holomorphic inside the circle K and satisfying the boundary condition (1.7), can be represented in the form

$$w(u) = v_0 \frac{8b}{\pi} \frac{u(1+u^2)\cos \sigma_1}{(1+u^2)^2 + 4u^2 \cos^2 \sigma_1} \left[\ln \frac{i-u}{i+u} - \frac{u - u^{-1}}{i\alpha - \frac{1}{i\alpha}} \ln \frac{1-\alpha}{1+\alpha} \right] \tag{1.8}$$

where

$$\alpha = \sqrt{1 + \cos^2 \sigma_1} - \cos \sigma_1$$

To determine the unknown value of the angle we shall make use of the equality $\lim \partial\psi/\partial z = v_0$ as $\lambda \to 1$. Extracting the imaginary part of (1.8) and differentiating it, we obtain for the determination of σ_1 the equation

$$\ln \frac{1 + \sqrt{1 + \cos^2 \sigma_1}}{\cos \sigma_1} = \sqrt{1 + \cos^2 \sigma_1}$$

which has the unique solution $\cos \sigma_1 = 0.663$. With this the ordinate of the point of separation P will be equal to $z = 0.92\ b$.

In order to determine the impulse force and the moment which act on the ellipse during the time of the impact, we shall find the potential ϕ, after extracting the real part from the characteristic function w:

$$\varphi = v_0 \frac{2}{\pi} \frac{u(u^2+1)\cos \sigma_1}{(1+u^2)^2 + 4u^2 \cos \sigma_1} \left[\ln \tan \left(\frac{\pi}{4} - \frac{\sigma}{2} \right) + \sin \sigma \right] \tag{1.9}$$

Using Formula (1.1) and carrying out the integration in the u-plane, we obtain

$$\begin{aligned} J_y &= -\rho \frac{2b^2}{\pi} v_0 \sin^2 \sigma_1 \\ J_z &= -\rho a b v_0 \left[\ln \frac{1 + \sqrt{1 + \cos^2 \sigma_1}}{\cos \sigma_1} - (1 + \cos^2 \sigma_1)^{-1/2} \right] \\ M &= \frac{1}{3} \rho b (b^2 - a^2) v_0 (1 + \cos^2 \sigma)^{-3/2} \end{aligned} \tag{1.10}$$

Hence, as $a \to 0$, for which the ellipse reduces to a flat plate, we obtain the formula given in [1].

For the virtual mass due to the impact on an incompressible fluid in the case of a horizontal impact we will have

$$\lambda_{22} = 0.56\rho\frac{2b^2}{\pi}, \qquad \lambda_{23} = 0.598\rho\frac{2ab}{\pi}, \qquad \lambda_{24} = -0.195\rho b(b^2 - a^2) \quad (1.11)$$

2. To verify the onset of separation of the fluid from the surface of the ellipse, a special experiment was carried out to measure the virtual mass along the free surface of the fluid.

The experimental body was a cut-off elliptical cylinder hanging on a long pipe from a welded frame.

The linear dimensions of the pipe are large compared to the dimensions of the body; therefore, the motion in the first moment after the impact of the weight of the pendulum on the elliptical cylinder (Fig. 2) can be

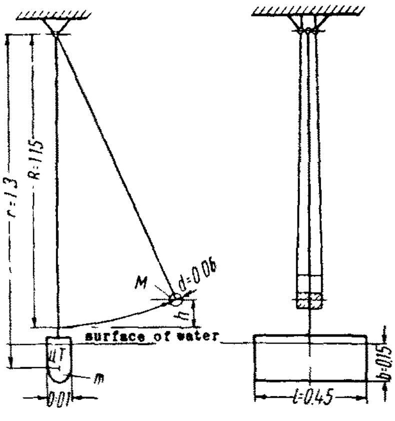

FIG. 2.

considered to be horizontal. This permits the equation for the momentum of the system before and after the impact to be written in the following form:

$$\begin{aligned} I_M\omega_{01} &= I_M\omega_1 + I_m\omega_2 \qquad (\text{in air}) \\ I_M\omega_{01} &= I_M\omega_1{}^1 + (I_m + I_\lambda)\,\omega_2{}^1 \quad (\text{in water}) \end{aligned} \qquad (2.1)$$

Here ω_{01} is the angular velocity of the pendulum weight before impact, ω_1, $\omega_1{}^1$ are the angular velocities of the pendulum weight after impact in air and in water, ω_2, $\omega_2{}^1$ are the angular velocities of the elliptical cylinder after impact in air and in water, and I_M, I_m, I_λ are the moments of inertia of the pendulum weight, the elliptical cylinder and the added mass of the water due to the impact.

The distance R from the axis of suspension to the center of gravity of the pendulum weight of mass M and the distance r from the axis of suspension to the center of gravity of the elliptical cylinder of mass are considerably larger than the dimensions of the colliding bodies; therefore, the proper moments of inertia may be neglected and it may be assumed that

$$I_M = MR^2, \qquad I_m = mr^2, \qquad I_\lambda = \lambda_{22}r^2 \tag{2.2}$$

The error associated with this is no more than 1%. We shall transfer from angular velocities to linear velocities, measuring the linear velocities at the point of collision of the bodies.

A second relation for the impact is written in the form

$$V_2 - V_1 = V_2{}^1 - V_1{}^1 = KV_{01} \tag{2.3}$$

Here V_{01} is the velocity of the pendulum weight before impact, V_1, $V_1{}^1$ are the velocities of the pendulum weight after impact in air and in water, V_2, $V_2{}^1$ are the velocities of the elliptical cylinder after impact in air and in water, and K is a constant, called the restoration coefficient of impact, which depends only on the material of the colliding bodies.

Using the relations (2.1)-(2.3), an expression for the virtual mass of water due to the impact can be presented in the form

$$\lambda_{22} = \left(\frac{V_2}{V_2{}^1} - 1\right)\left(m + M\,\frac{R^2}{r^2}\right) \tag{2.4}$$

Thus, knowing the velocity of the cylinder after impact in air and in water at the initial moment of the motion when the effect of the resistance of the water has not yet been felt, one can find the added mass λ_{22}.

The impact process was photographed with a motion picture camera with a photographic frequency of 150 frames per second. By reading the film the dependence of the displacement of the cylinder on the time due to the impact in water and in air was found. Because the construction was not sufficiently rigid, elastic vibrations of the body appeared, and for the calculation the mean dependence on time was taken, which, after differentiation, gave the velocity of the motion of the cylinder after impact in water and in air.

In the photographs (Fig. 3) are shown four successive stages of the impact of the cylinder on water, from which it is seen that separation of the fluid is to be observed on the back side of the cylinder (t = 0.0370 – 0.0637 sec) and that the experimental point of separation (z = 0.85 b) is close to that obtained theoretically (z = 0.92 b).

The velocities V_2 and V_2^1 are determined by graphical differentiation at $t = 0$ and the added mass of the water due to the impact of the elliptical cylinder on the water is obtained according to Formula (2.4): $\lambda_{22} = 0.368$ kg sec^2/*M*.

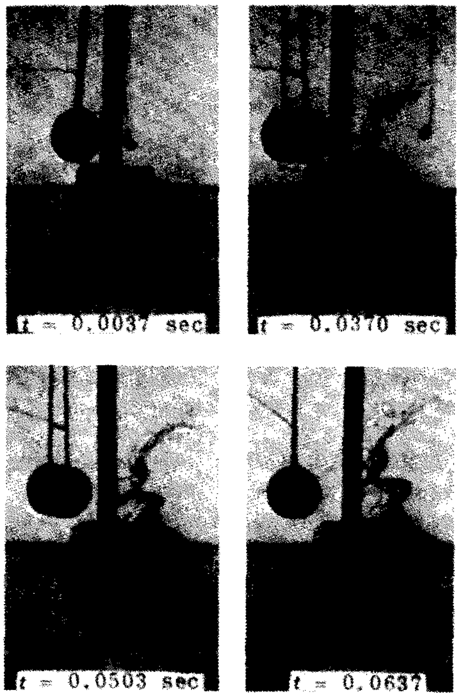

FIG. 3.

The value of the added mass, theoretically calculated according to Formula (1.11), is equal to 0.327 kg sec^2/*M*. The difference in the experimental and theoretical values of the added masses is not serious (13%) and to a considerable extent is accounted for by the errors in the experimental installation and method.

The described experiment confirmed the basic theoretical promise about the origin of the separation of the fluid from the surface of the ellipse.

The exact solution to the problem of the horizontal impact of an ellipse has shown that the approximate determination of the point of separation and of the impulsive forces and the moment on the basis of the solution of the problem of the non-separated impact of an ellipse leads to large differences.

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